On the forecasting horizon of seismicity models

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5th International Workshop on Statistical Seismology, Erice, 30 May – 6 June 2007

Introduction

- **Seismicity models often fitted and assessed based on instantaneous rate densities $\lambda(t,m,x,y)$, by maximising the log likelihood**

$$\ln L = \sum_{i=1}^{n} \ln \lambda(t_i, m_i, x_i, y_i) - E(N)$$

of the earthquake catalogue $\{(t_i,m_i,x_i,y_i),\ i=1,\ldots,n\}$ in a region.

- **Here, $\lambda(t,m,x,y)$ is typically evaluated using all prior information, however close to time t.**

Practical forecasting

In practice there is some future time-period of interest e.g.,

- next day
- next year
- next five years

Forecast must be made using only information available before the beginning of the period.

This may affect the performance of a model

Forecasting horizon

- **Let F_t denote information available up to but not including time t. Then define**

$$\lambda^0(t,m,x,y) = \lambda(t,m,x,y \mid F_t)$$

and

$$\lambda^\tau(t,m,x,y) = \lambda(t,m,x,y \mid F_{t-\tau})$$

- **At any time, λ^τ looks ahead to a forecasting horizon τ beyond the present.**

Information rate at a forecasting horizon

- A model's performance can be quantified by its *information rate*, $\Delta \ln L / n$, where

$$\Delta \ln L = \ln L - \ln L_0$$

where L and L_0 denote the likelihood under the model of interest and a reference model.

- Let the likelihood of the conditional intensity λ_τ with forecasting horizon τ be denoted $L(\tau)$. The information rate at forecasting horizon τ is then $\Delta \ln L(\tau) / n$, where

$$\Delta \ln L(\tau) = \ln L(\tau) - \ln L_0$$

Visibility at a forecasting horizon

$$\text{Visibility at forecasting horizon } \tau = \frac{\Delta \ln L(\tau)}{\Delta \ln L(0)}$$

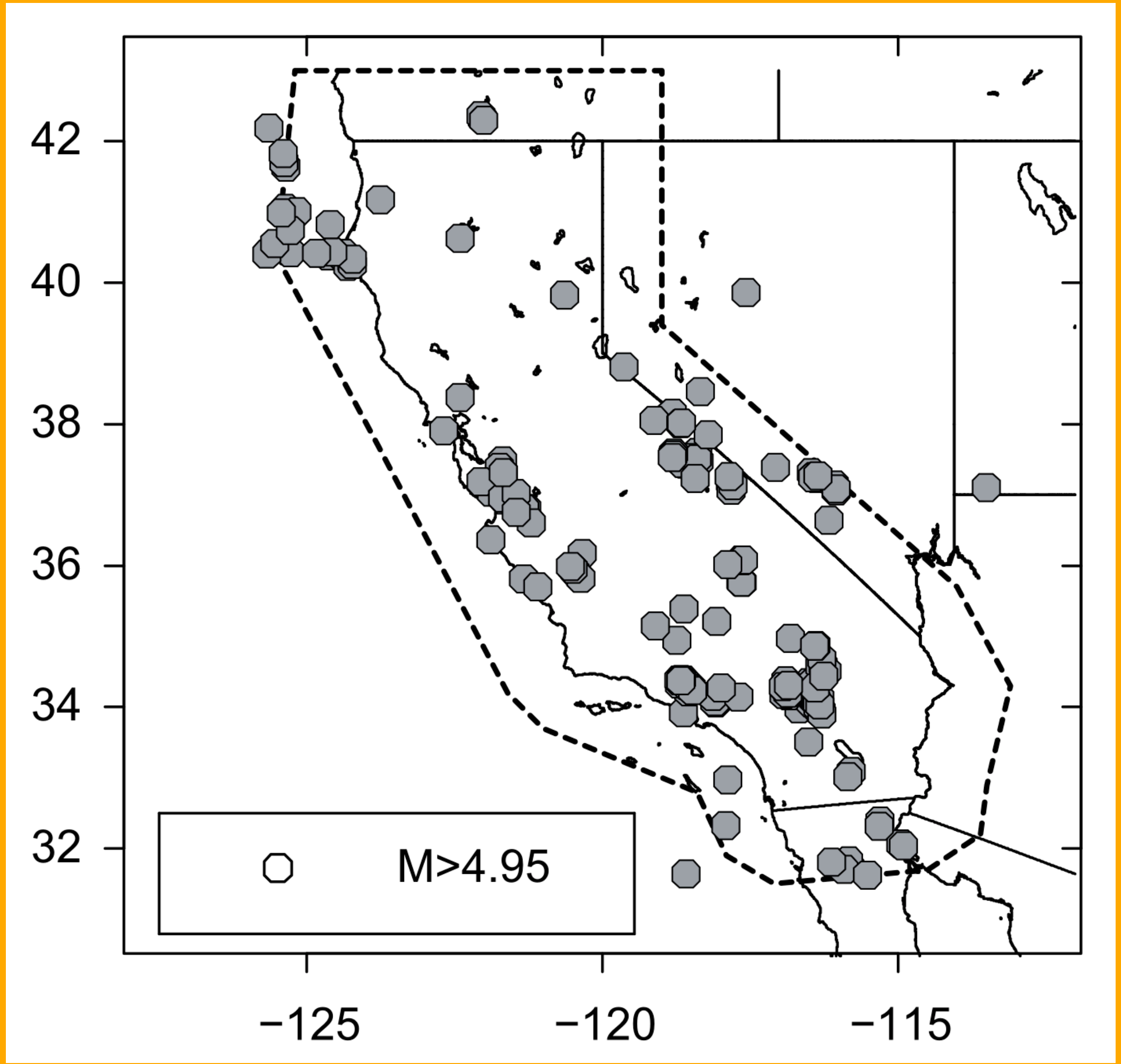
50% visibility horizon

$$\frac{\Delta \ln L(\tau_{50})}{\Delta \ln L(0)} = 0.5$$

Examples

- **SUP – stationary and spatially uniform Poisson model: static reference model.**
- **PPE – proximity to past earthquakes: quasi-static spatially varying reference model which updates as each new major earthquake occurs**
- **ETAS – time-space epidemic type aftershock model: short-term forecasting model which updates as each new earthquake occurs**
- **EEPAS – every earthquake a precursor according to scale: long-range forecasting model which updates as each new earthquake occurs**

Models were fitted to RELM test region, 1984-2004



m_c: Magnitude threshold for forecast

m_0: Magnitude threshold for data (EEPAS and ETAS)

Catalogue data from 1932 used.

PPE model

$$\lambda^{\tau}_{PPE}(t,m,x,y) = f_0(t)g_0(m)h_0(t,x,y)$$

where

$$f_0(t) = 1/(t - \tau - t_0),$$

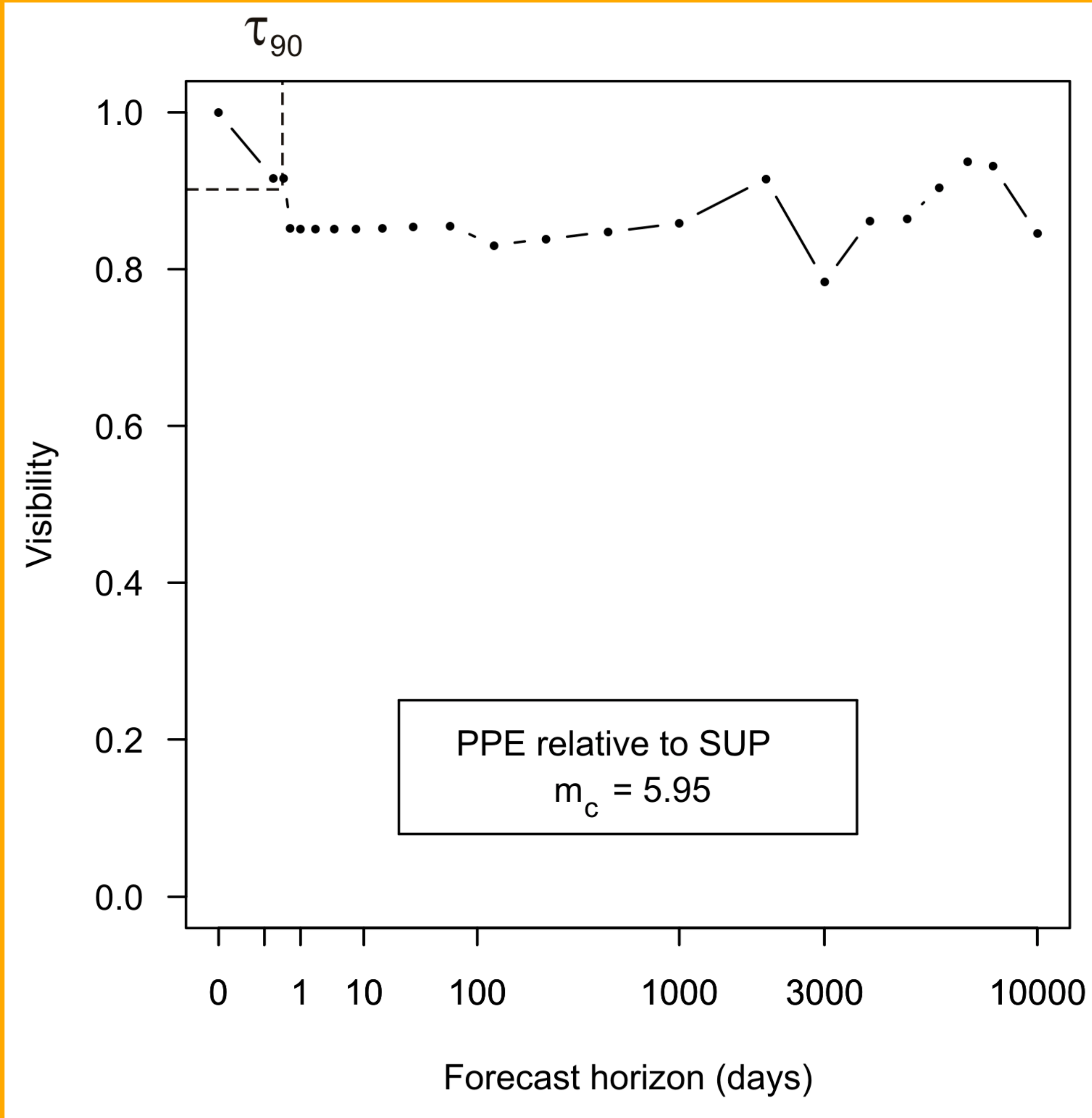
$$g_0(m) = \beta \exp\left[-\beta(m - m_c)\right] \quad (m > m_0),$$

$$h_0(t,x,y) = \sum_{t_i = t_0}^{t-\tau} h_{0i}(r_i), \quad \text{where}$$

$$h_{0i}(r_i) = a(m_i - m_c)\frac{1}{\pi}\left(\frac{1}{d^2 + r_i^2}\right) + s$$

Refs : Jackson and Kagan (1999), Rhoades and Evison (2006)

Visibility of PPE model relative to SUP



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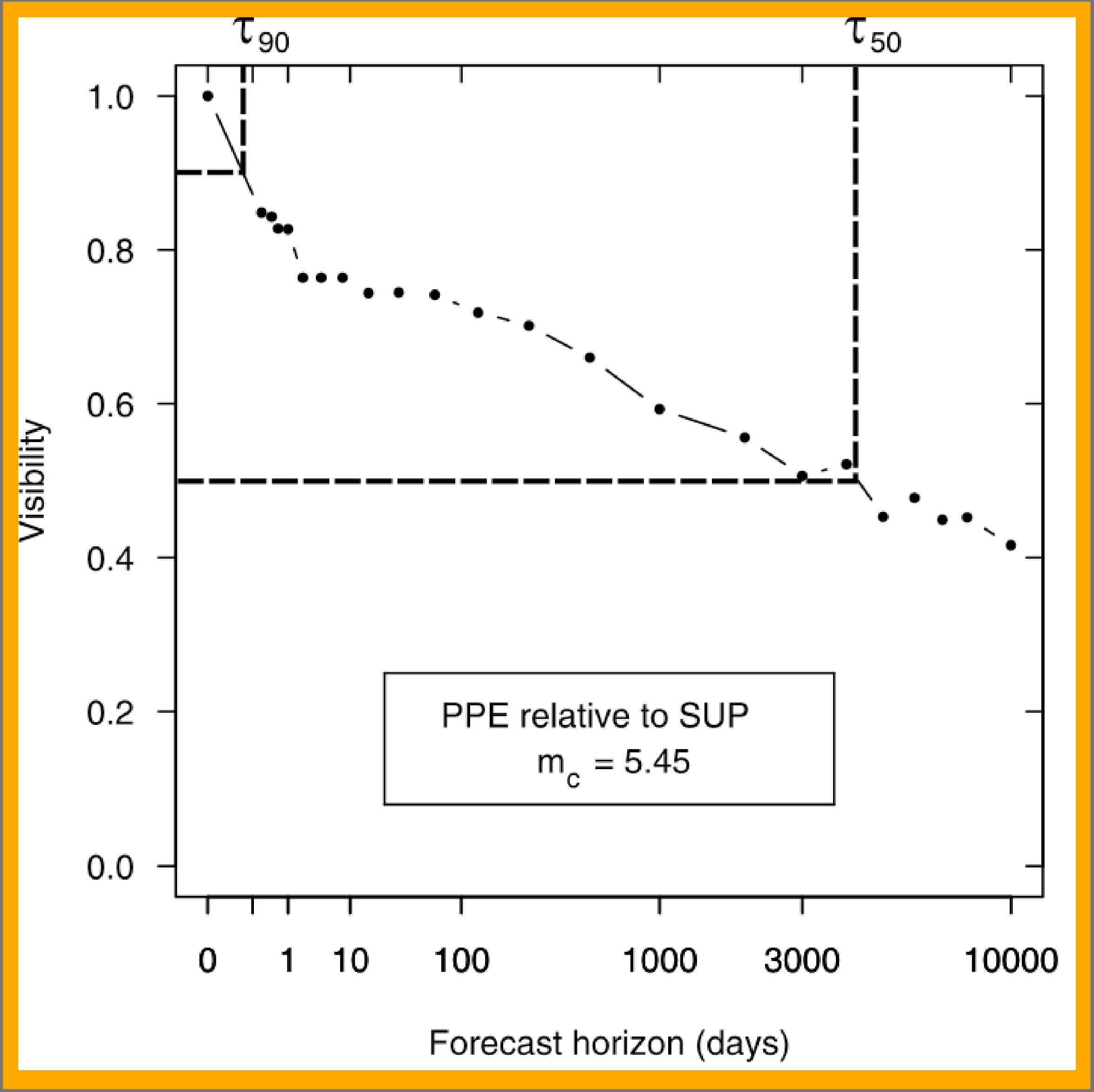

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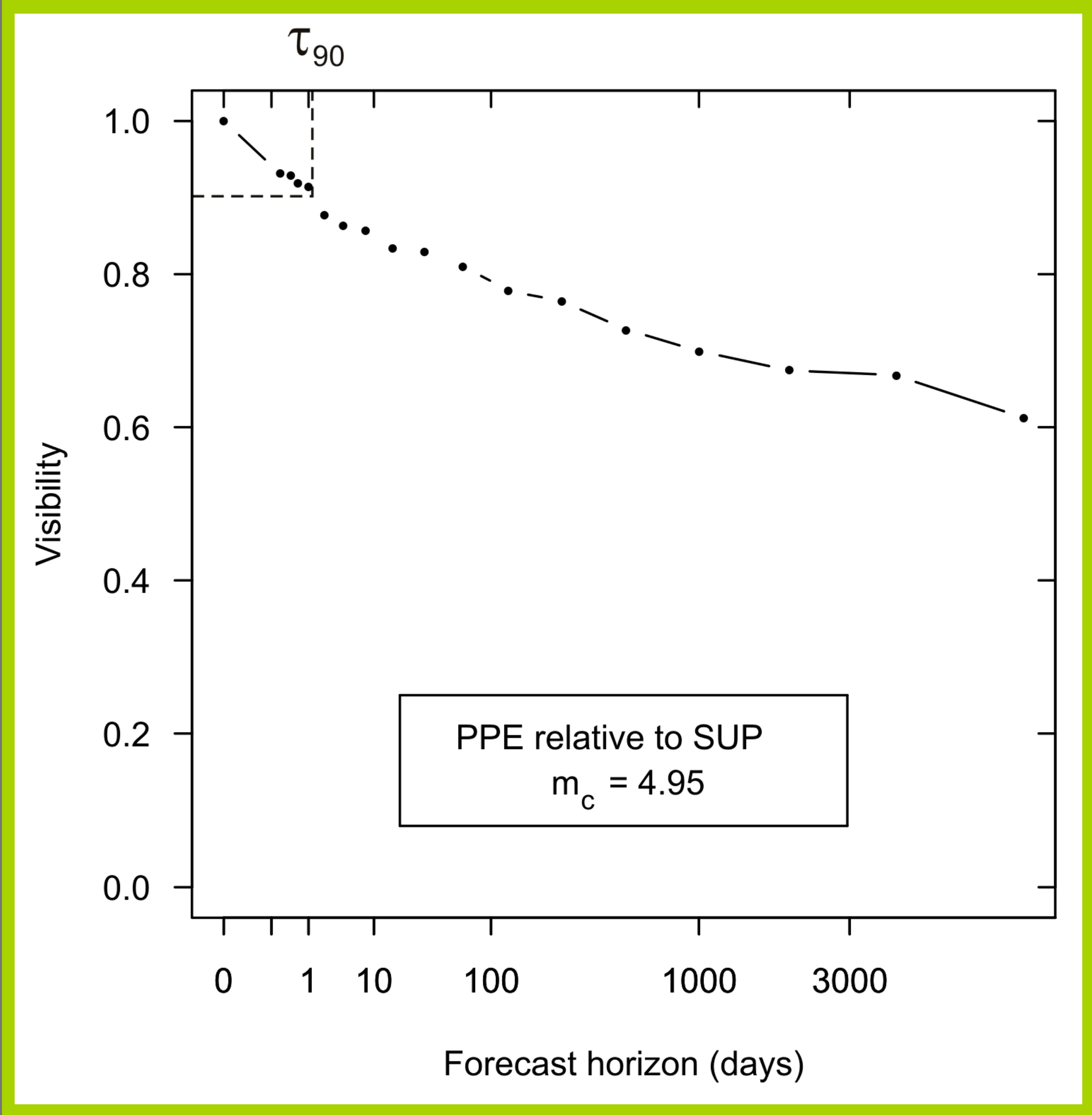


ETAS model

$$\lambda^{\tau}_{ETAS}(t,m,x,y) = \nu\lambda^{\tau}_{PPE}(t,m,x,y) + \sum_{t_i=t_0}^{t-\tau} f_{1i}(t)\,g_{1i}(m)\,h_{1i}(x,y)$$

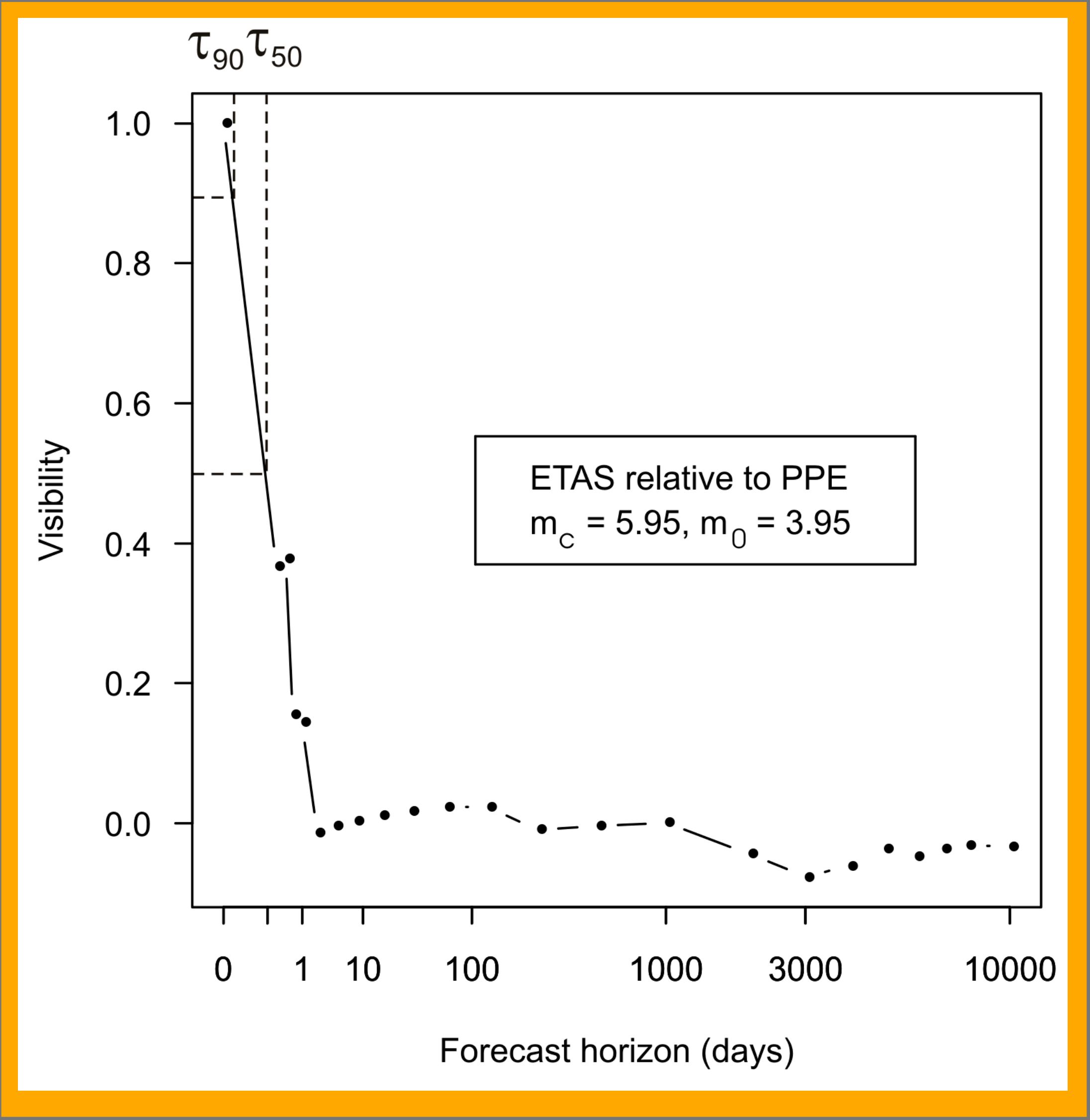
where

$$f_{1i}(t) = H(t-t_i)\frac{p-1}{(t-t_i+c)^p},$$

$$g_{1i}(m) = \beta\exp\left[-\beta(m-m_i)\right],$$

$$h_{1i}(x,y) = \frac{1}{2\pi\sigma^2 10^{m_i}}\exp\left[-\frac{(x-x_i)^2+(y-y_i)^2}{2\sigma^2 10^{m_i}}\right].$$

Visibility of ETAS relative to PPE at τ = 80 days



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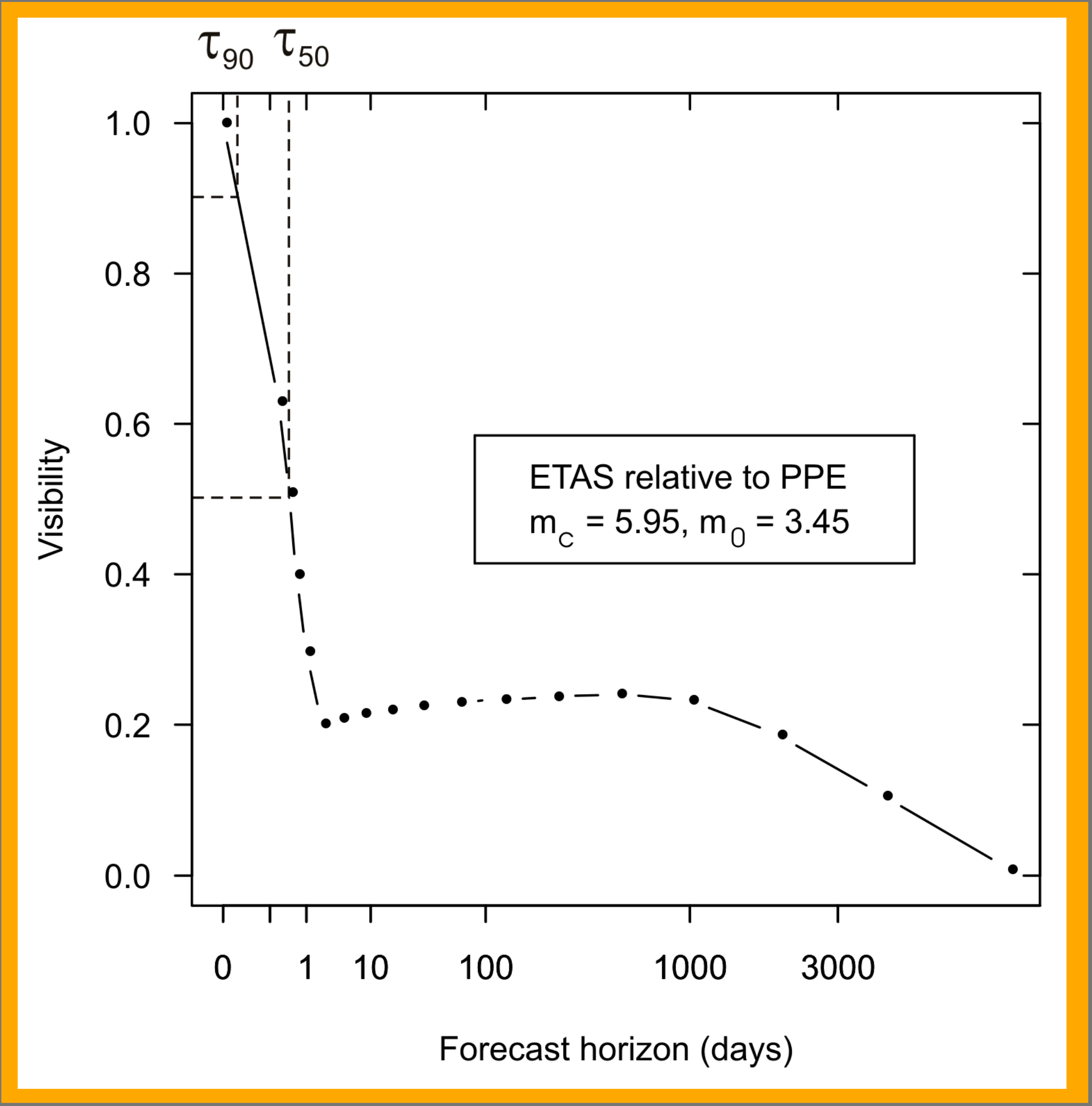

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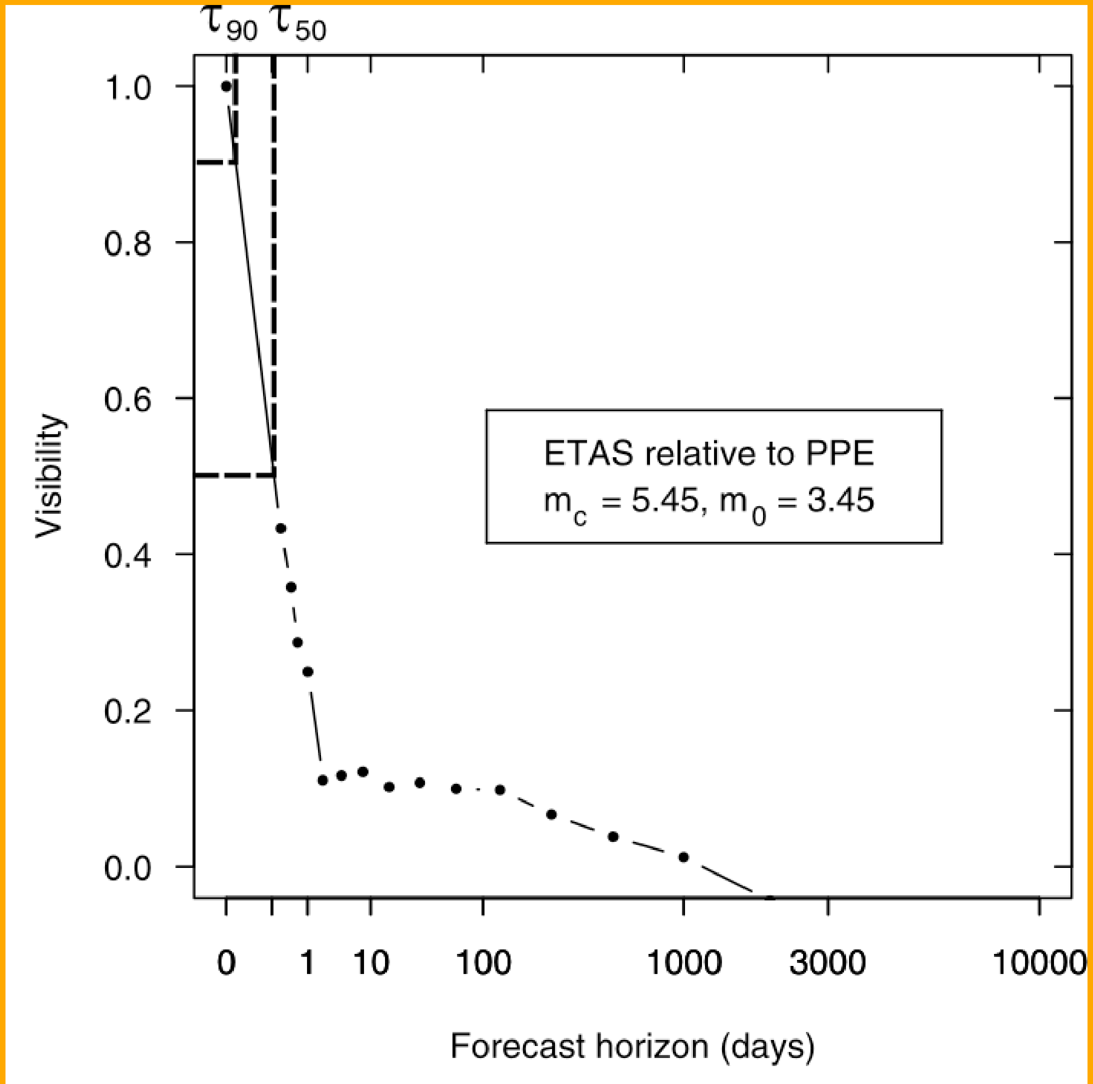

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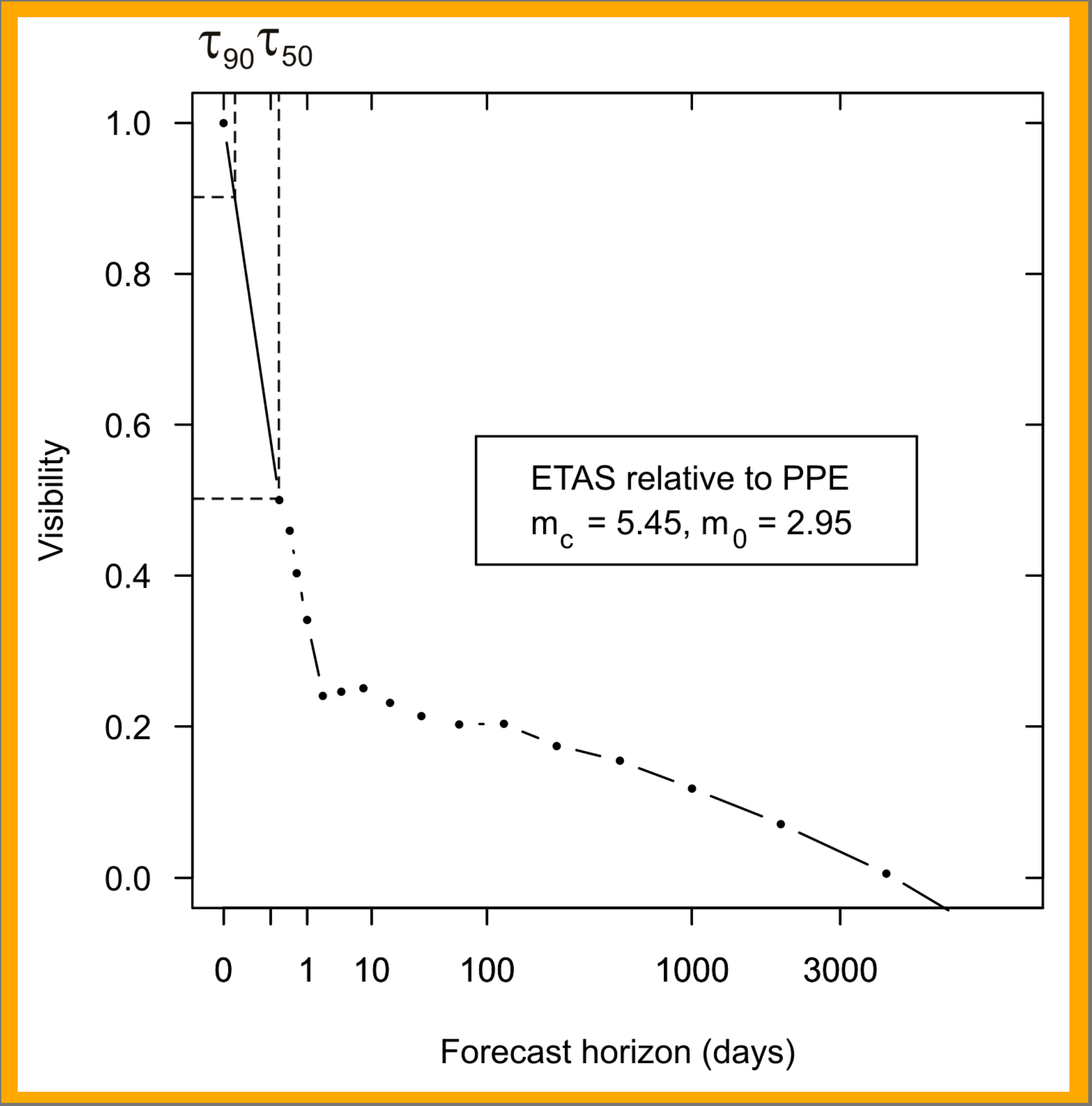

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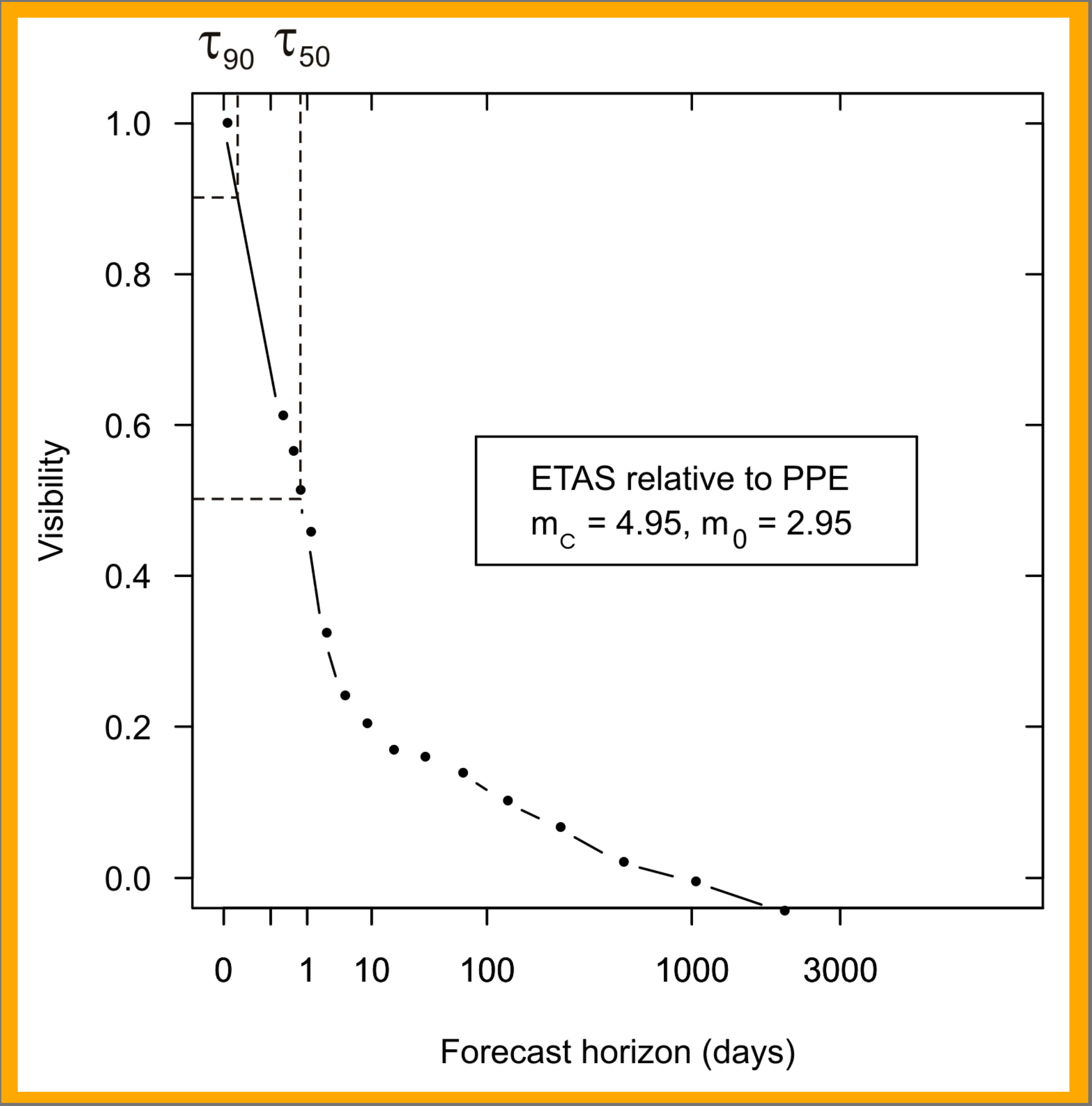


EEPAS model

$$\lambda^{\tau}_{EEPAS}(t,m,x,y) = \mu\lambda^{\tau}_{PPE}(t,m,x,y) + \sum_{t_i=t_0}^{t-\tau}\eta(m_i) f_{2i}(t) g_{2i}(m) h_{2i}(x,y)$$

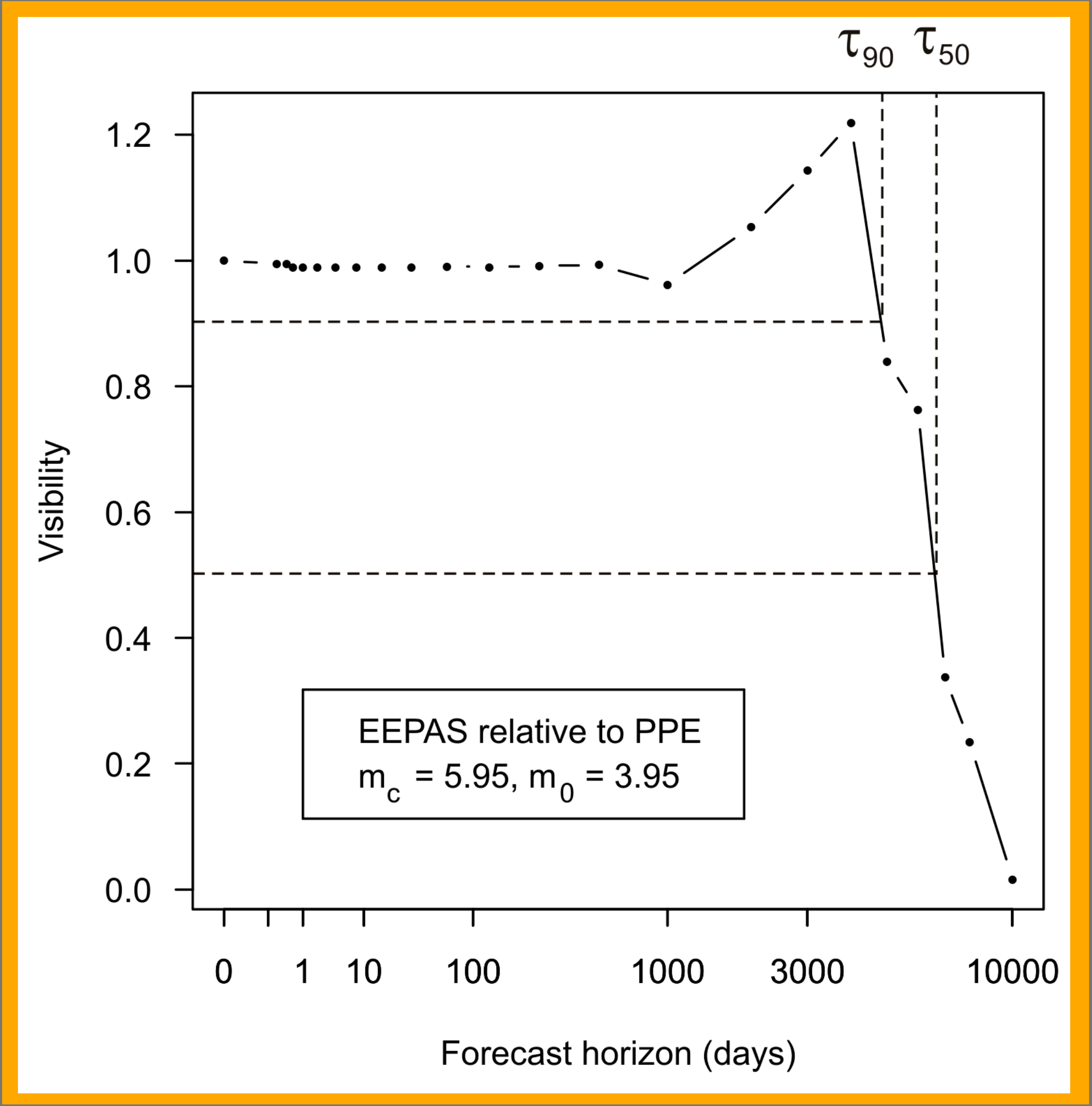
where $\eta(m_i)$ is a normalizat ion function, and

$$f_{2i}(t) = \frac{H(t-t_i)}{\sigma_M \ln(10)\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\log(t-t_i) - a_T - b_T m_i}{\sigma_T}\right)^2\right],$$

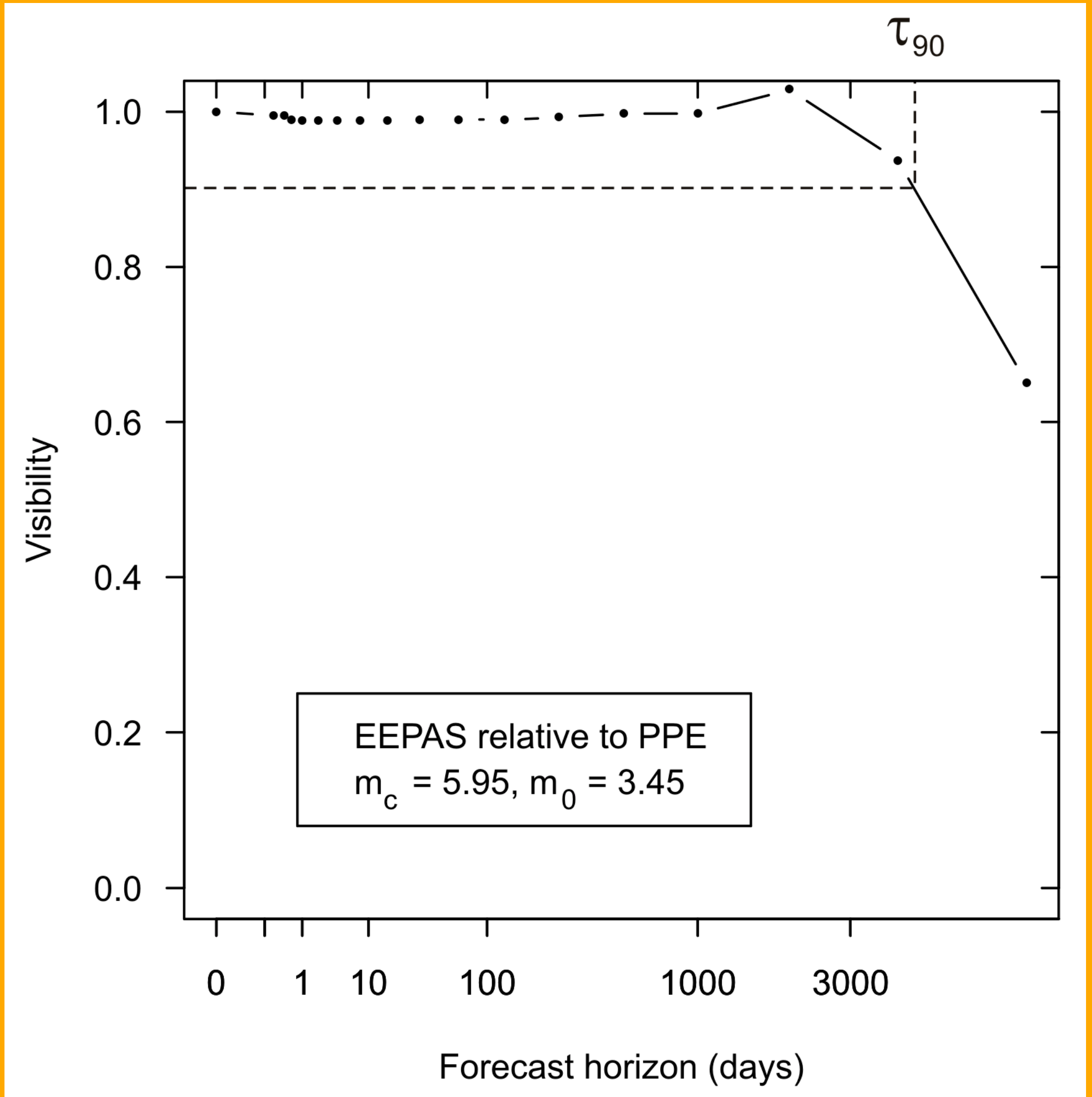
$$g_{2i}(m) = \frac{1}{\sigma_M\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{m - a_M - b_M m_i}{\sigma_m}\right)^2\right],$$

$$h_{2i}(x,y) = \frac{1}{2\pi\sigma_A^2 10^{b_A m_i}} \exp\left[-\frac{(x-x_i)^2 + (y-y_i)^2}{2\sigma_A^2 10^{b_A m_i}}\right].$$

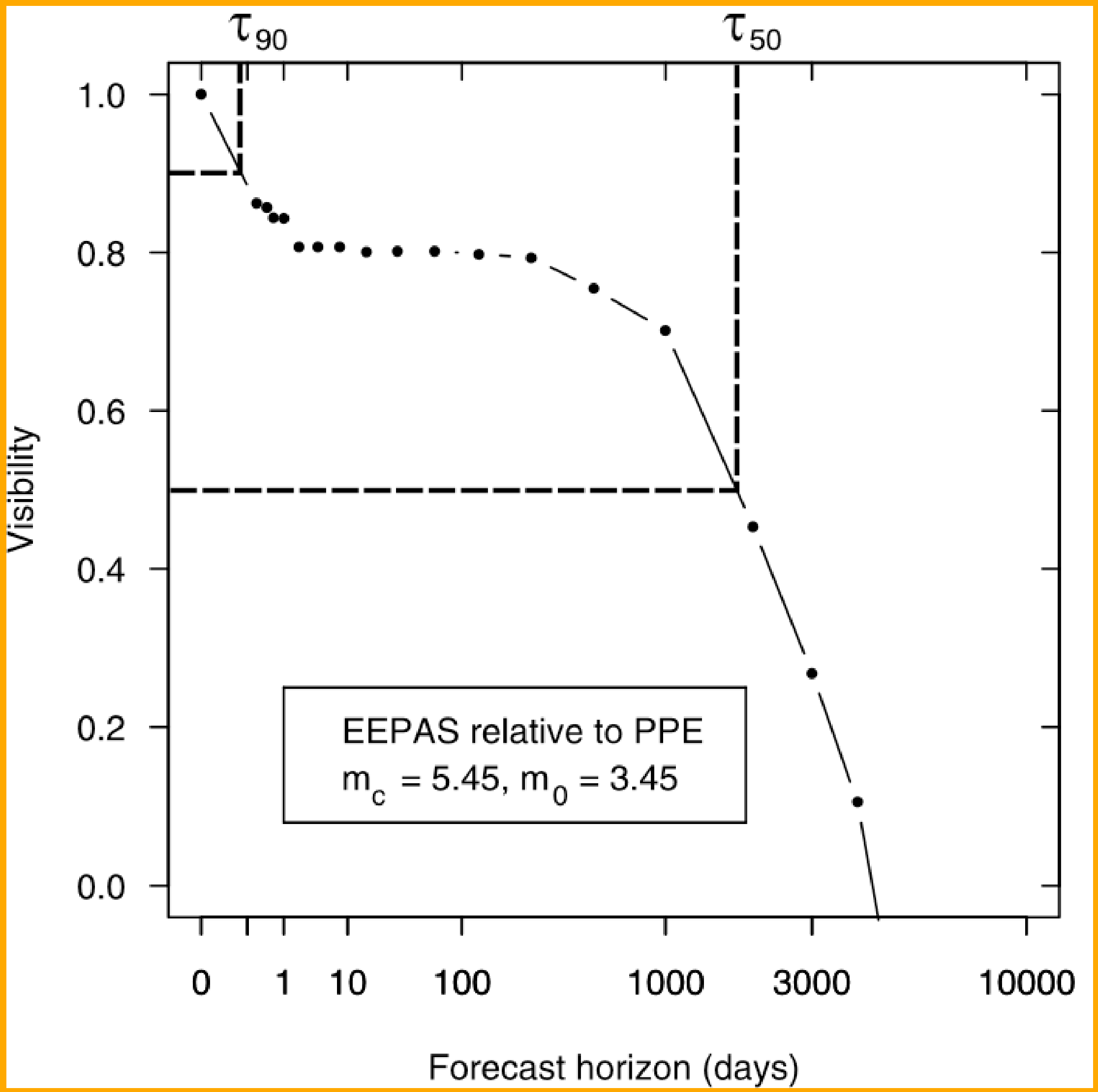
Visibility of EEPAS relative to PPE at τ = 80 days



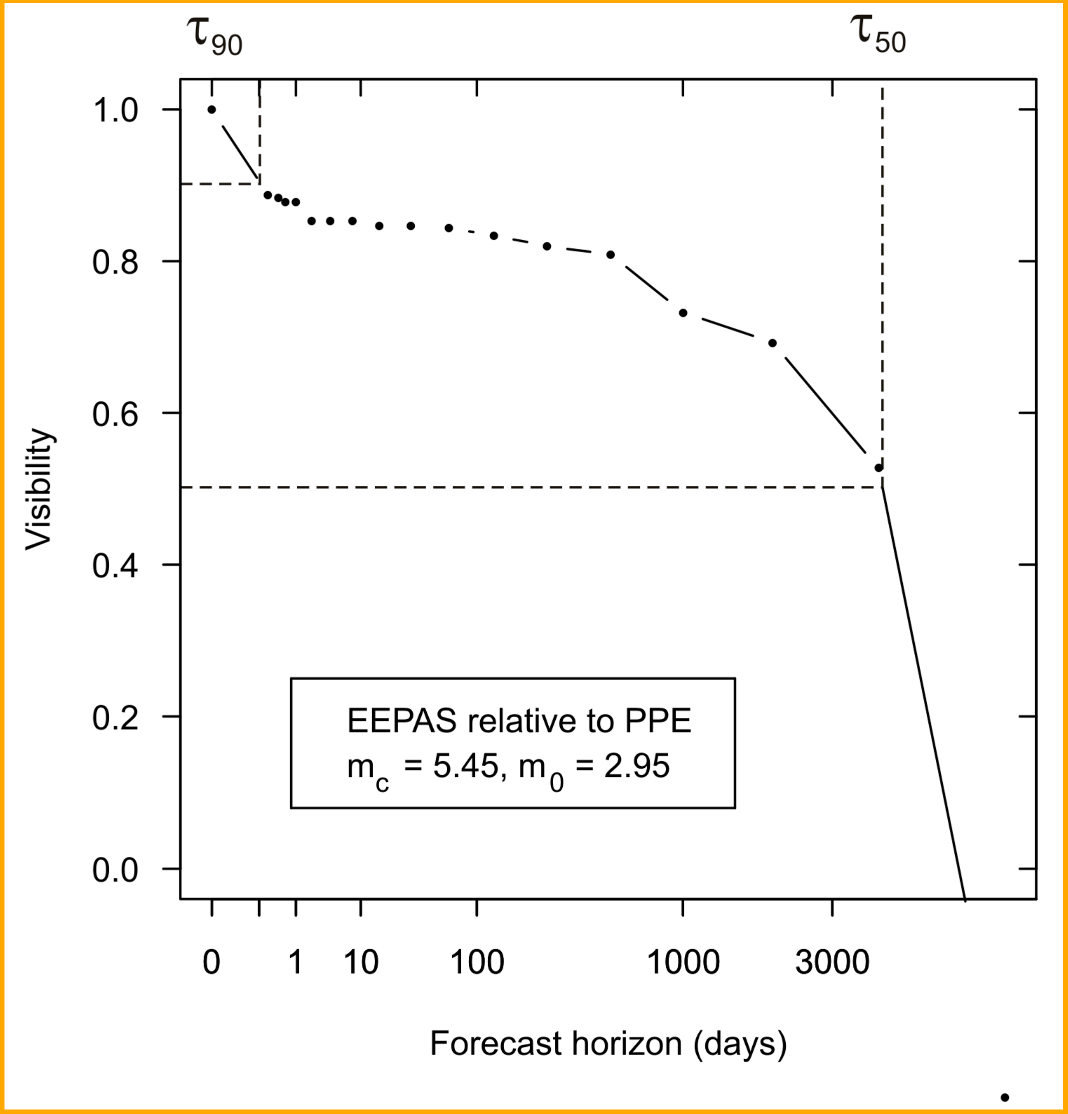
Visibility of EEPAS relative to PPE at τ = 80 days



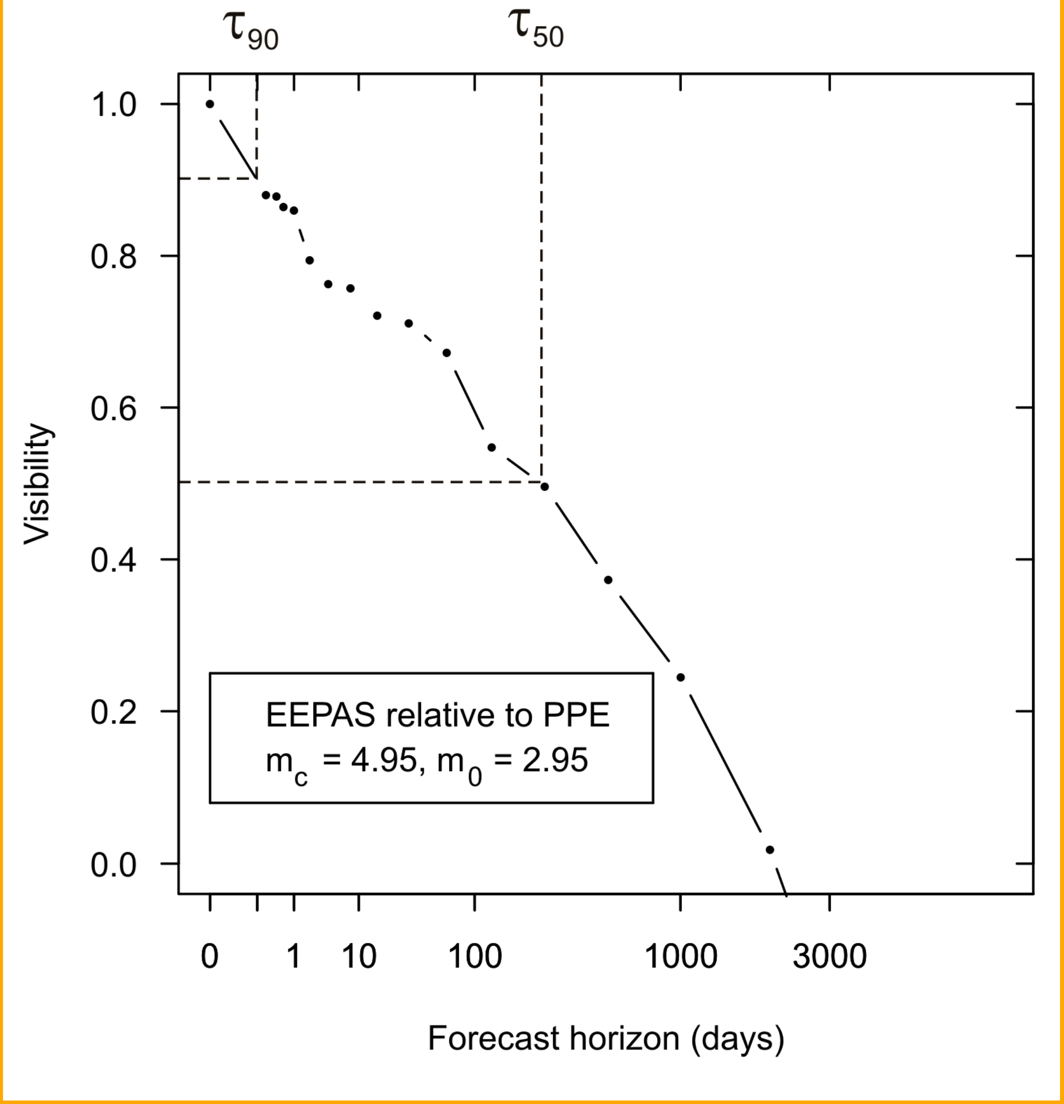
Visibility of EEPAS relative to PPE at τ = 80 days



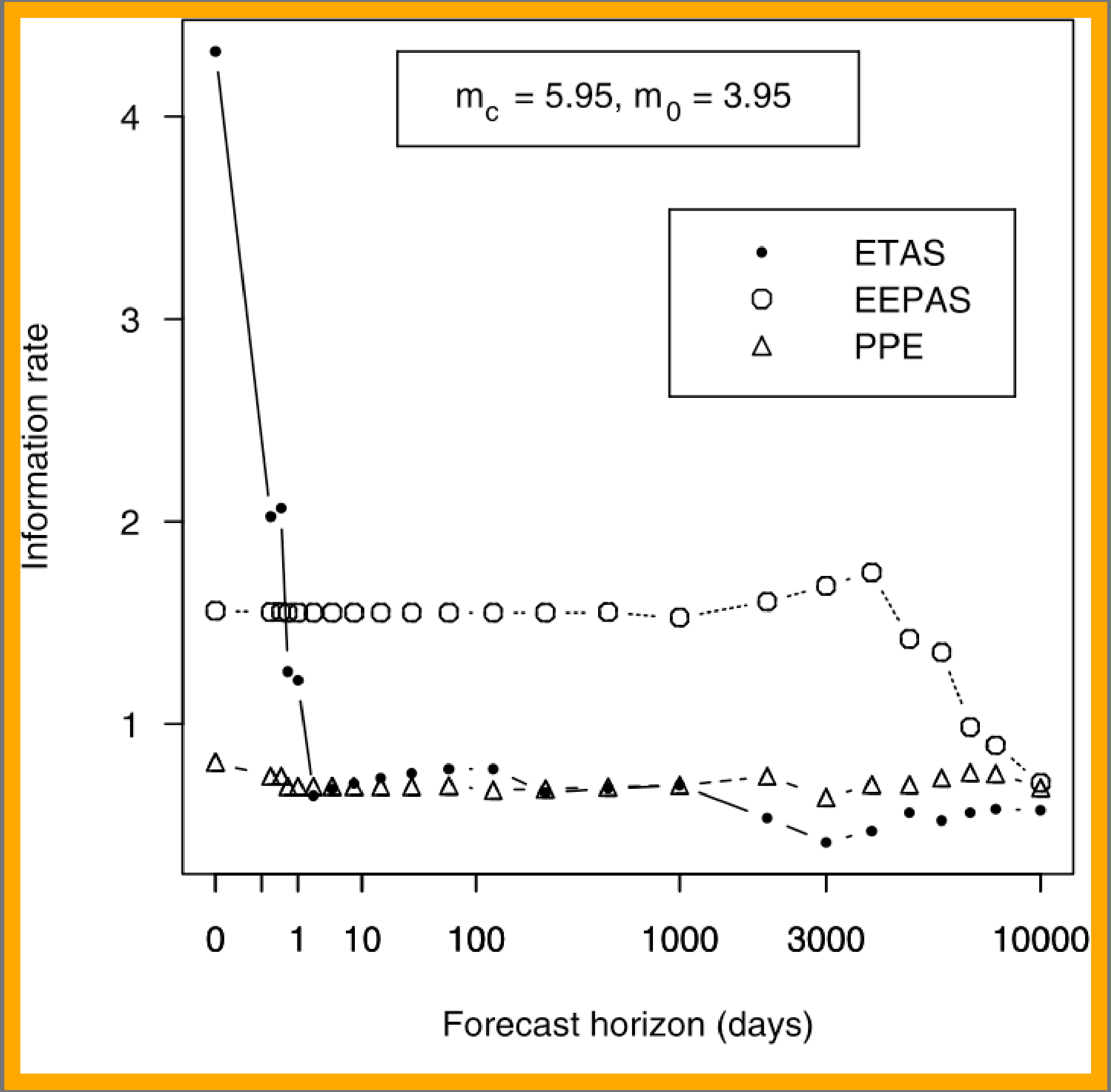
Visibility of EEPAS model relative to PPE at τ = 80 days



Visibility of EEPAS relative to PPE at τ = 80 days



Comparison of information rates (relative to SUP)



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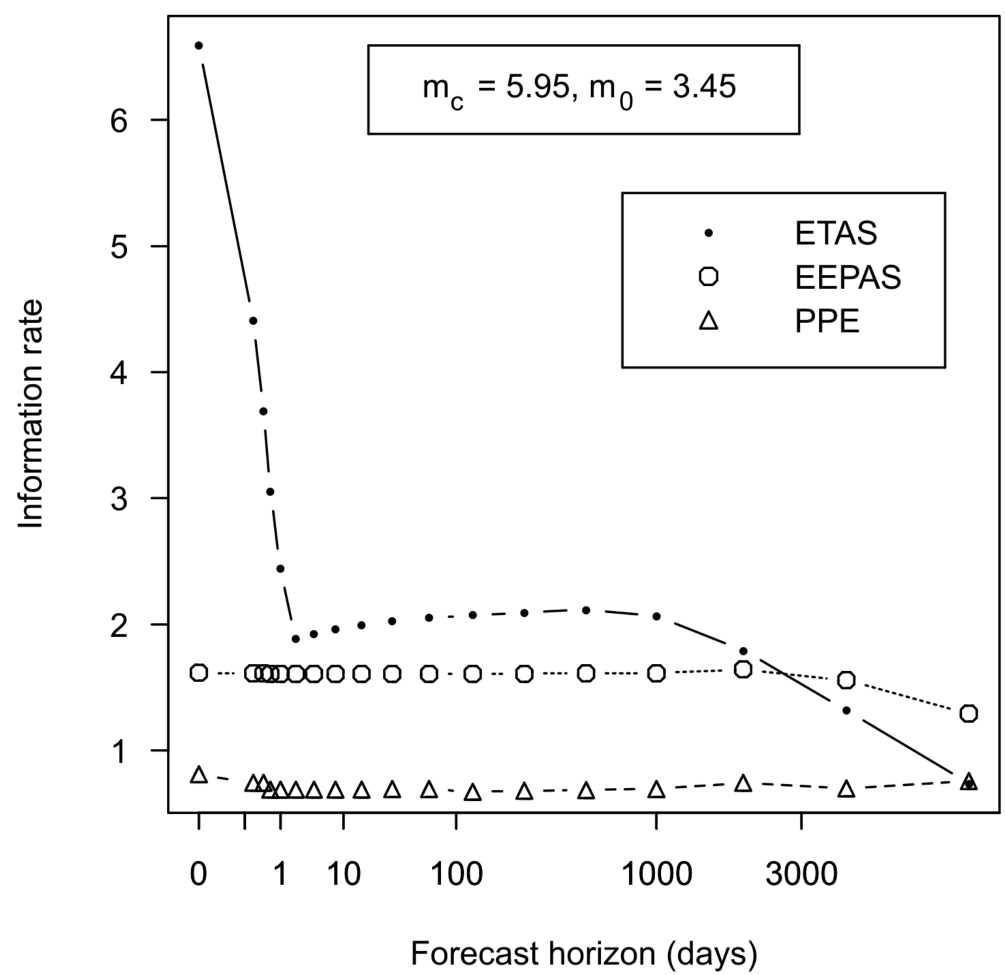

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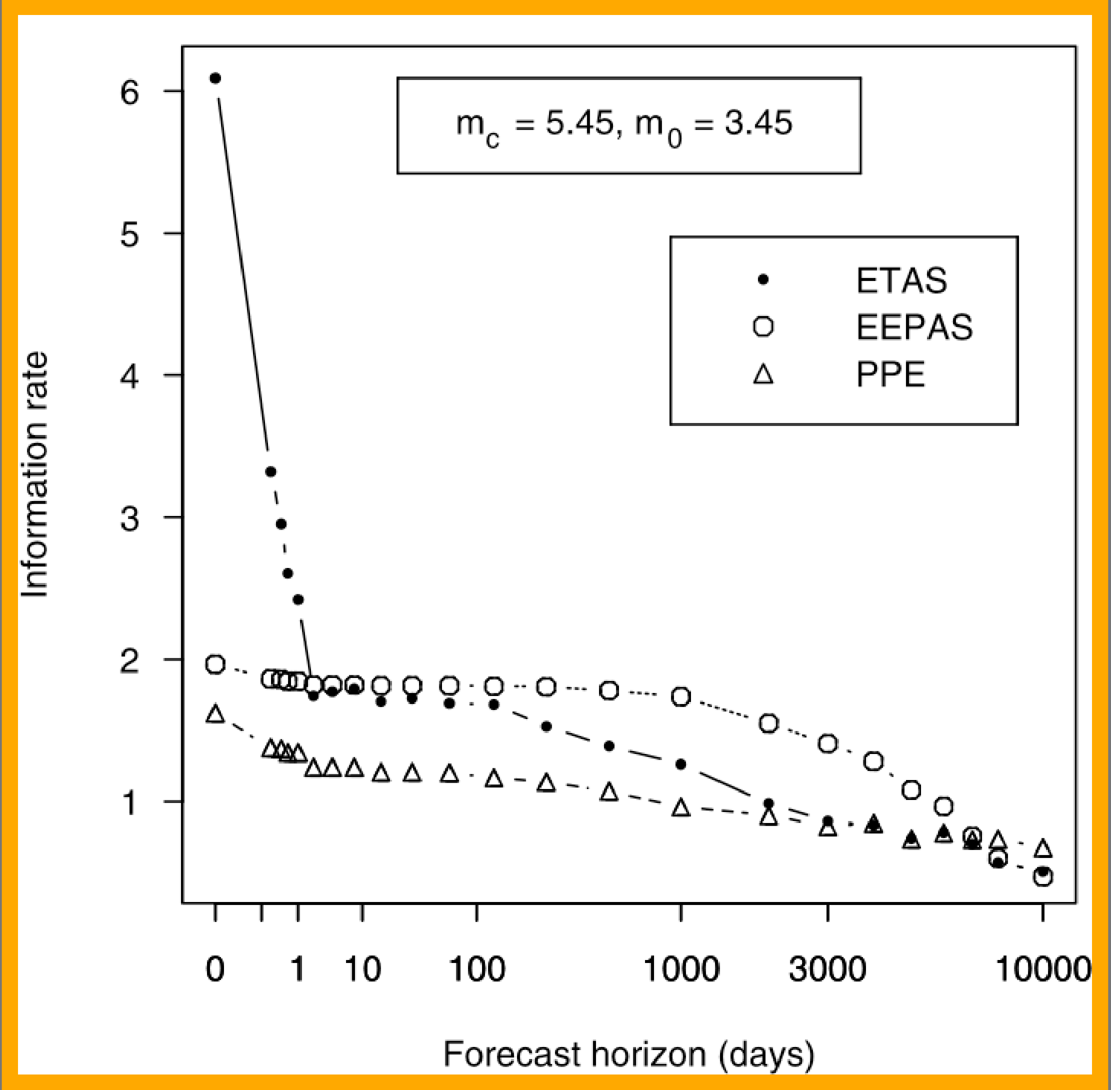

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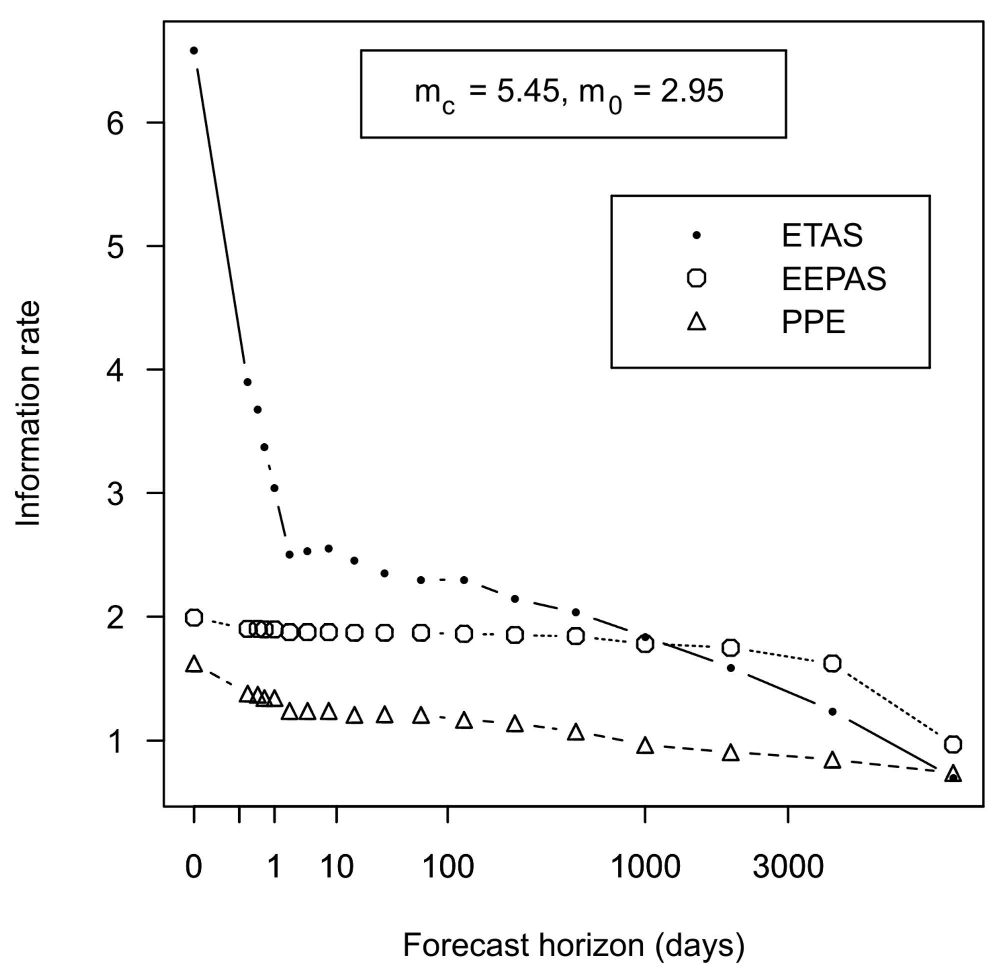
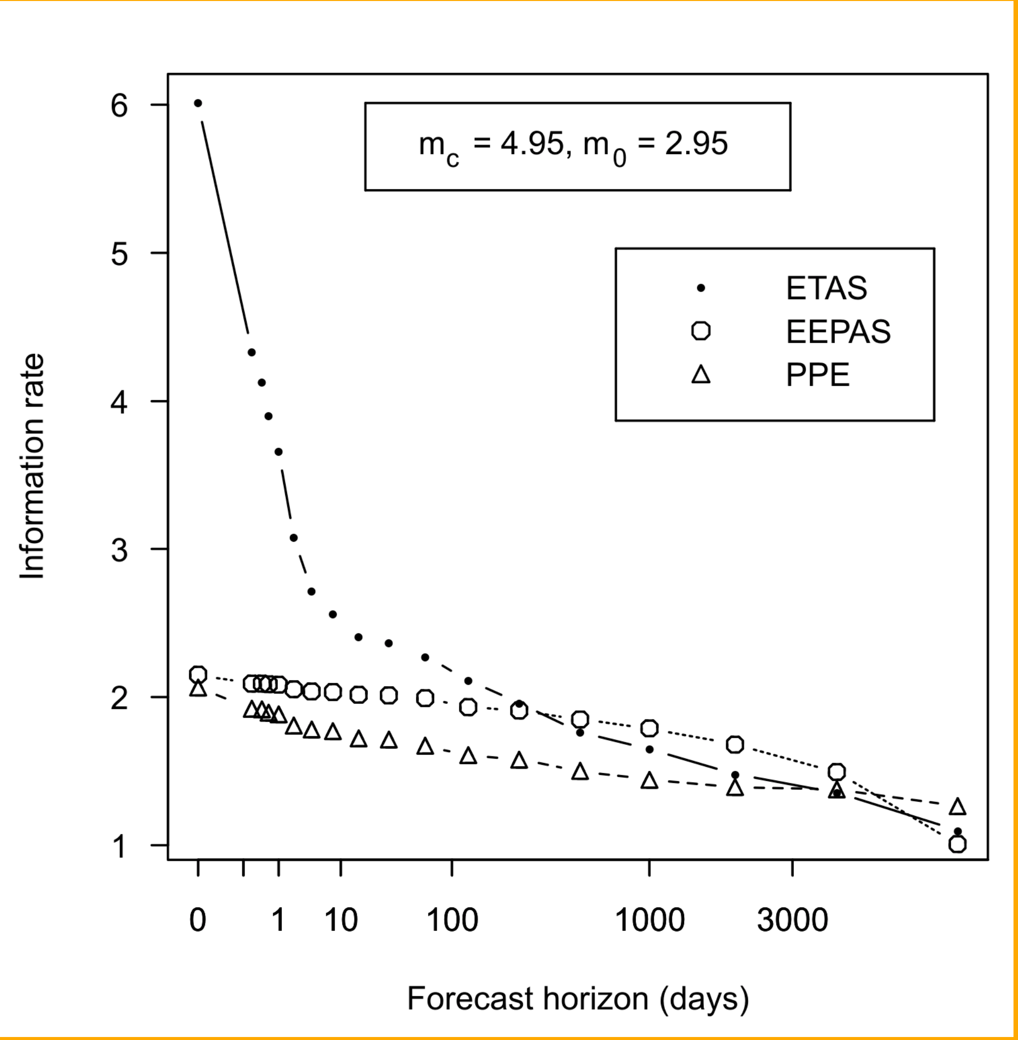


Comparison of information rates (relative to SUP)



Conclusion

- Analysis of the visibility of a model at different forecast horizons is a useful exploratory tool that can help us understand and improve existing seismicity models.
- It shows at what forecast horizons a model has predictive capability, and so is a guide to the usefulness of the model in practical forecasting.
- Comparison of information rates of different models as a function of forecast horizon may help us to construct optimal composite models for forecasting in a time-window of given length.